Analysis of Tipping Points in Social Networks for Diffusion of Innovations

Seulki Lee
KAIST
sklee19@kaist.ac.kr

Hyuna Kim
KAIST
hyunak@kaist.ac.kr

Kyomin Jung
KAIST
kyomin@kaist.edu

Tipping point phenomena (events that had rarely observed becomes suddenly common) for diffusion of innovations have received huge attention from academia and industry [4,6,12]. Understanding tipping point phenomena has numerous applications including viral marketing and minimizing the spread of contamination. Depending on the characteristics of the information and social network structures, the information either cascades globally or terminates quickly. For example, sometimes new technologies become widespread over the network (a global cascade), but in some cases they simply disappear in a short time. In this work, we identify conditions for the occurrence of tipping points for general classes of network structures and provide a novel proof for its correctness.

Various models of information spreading have been studied. These models are established based on the common assumption that the neighbors play significant roles for the spread of information. The SIR (Susceptible-Infected-Recover) model is one of those popular models applied to the cases when accepting the information requires low costs, such as the epidemics of contagious diseases [1,2,8]. Under the SIR model, some sufficient conditions for a global cascade have been studied [2,3,9]. On the other hand, for the diffusion of new technologies or innovations which requires relatively high costs to adopters, the linear threshold model is widely used [7,12,13]. However, general conditions for a global cascade under the linear threshold model are known for restricted cases.

In the linear threshold model, individuals make their decisions based on the decisions of their neighbors. Each node has its own threshold value and if the fraction of neighbors who have already adopted the innovation is greater than the threshold, it will adopt the innovation. The mechanism of this model is originated from the utility maximization of individuals in game theory.

A tipping point is defined as the number of initial adopters x so that the cascade size becomes suddenly large as the number increases from x to $x + \delta$ for a small δ. Under the linear threshold model, the mechanism how a tipping point arises in a complete graph is well known [5]. When the thresholds of all nodes are homogeneous, the average cascade size and the number of initial adopters that triggers a global cascade have been predicted in the case of Erdös-Rényi random graph networks [13]. However, it is known that distributions of thresholds usually follow diverse unimodal distributions such as the normal distribution [10,11].

In this work, we consider any distributions of thresholds and assume that each node takes its threshold value from the distribution independently. We first analyze that in a social networks including Facebook and Myspace, tipping points occur almost always if certain conditions on the distribution of thresholds are met. We provide a novel proof that under those conditions, a tipping point occurs almost surely for any graphs whose nodes' degrees are $\omega(\log n)$, where n is the number of nodes. Our proof can be applied to any distributions of thresholds such as the uniform, the normal and homogeneous distributions, and it works for any class of graphs with reasonably high degrees. We also numerically analyze that in graphs having nodes with $O(\log n)$ degrees, the similar result holds.

Secondly, we conducted extensive experiments on real world social networks such as Facebook and Myspace, and synthetic network graphs including Erdös-Rényi random graphs, generalized random graphs with expected degree sequences, and scale-free networks generated by the preferential attachment process. We discover that tipping points indeed appear in these graphs if similar conditions on threshold distributions are met. In order to investigate properties of tipping points, we performed experiments on various network structures with regard to the network size, degree distributions and their community structures. We obtain strikingly similar results from several independent network graphs and conclude that even though some properties of network structures can affect tipping points to some extent, the distribution of thresholds are much more relevant to them.

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