Feasible rate allocation in wireless networks

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***Abstract*—Rate allocation is a fundamental problem in the operation of a wireless network because of the necessity to schedule the operation of mutually interfering links between the nodes. Among the many reasons behind the importance of *efficiently* determining the membership of an arbitrary rate vector in the feasibility region, is its high relevance in optimal cross layer design. A key feature in a *wireless* network is that links without common nodes can also conflict (secondary interference constraints). While the node exclusive model problem has efficient algorithms [8], it has long been known that this is a hard problem with these additional secondary constraints [1].**

However, wireless networks are usually deployed in geographic areas that do not span the most general class of all graphs possible. This is the underlying theme of this paper, where we provide algorithms for two restricted instances of wireless network topologies. In the first tractable instance, we consider nodes placed *arbitrarily* in a region such that (a) the node density is *bounded*, and (b) a node can only transmit or interfere with other nodes that are within a certain *limited* radius. We obtain a simple $(1-\varepsilon)$ polynomial-time approximation scheme for checking feasibility (for any $\varepsilon > 0$). The second instance considers the membership problem of an arbitrary rate-vector in the feasible set, where the nodes are distributed within a slab of fixed width (there are no density assumptions). Specifically, the results in [12] are shown to extend to a much more general class of graphs, which we call the (d_{min}, d_{max}) class of graphs, and this generalization is used to obtain a strongly polynomial time algorithm that decides membership of a rate-vector where the hosts are distributed within an infinite corridor with fixed cross-section.

I. Introduction

Wireless networks are inherently different from wireline networks due to the effects of *interference*: transmission of a wireless node can adversely affect other simultaneous transmissions among neighboring nodes. Interference is a major reason why the link-rates in a wireless network are not fixed independently – they are dependent on the other transmissions. This makes the determination of link-rates that can be allocated in a wireless networks a non-trivial problem.

A. Motivation

The problem of determining what link rates are feasible in a wireless network is a very important question. Here we present a motivating example from the perspective of cross-layer design.

Scheduling link transmissions in a wireless network so as to optimize one or more of the performance objectives (e.g. throughput, delay, energy) has been a topic of much interest over the past several decades. Tassiulas and Ephremides [19] characterized the capacity region of packetized constrained queueing systems, such as wireless networks, by means of a queue-length based scheduling policy. Effectively, their result implies that if a multi-hop wireless network is required to serve data at a rate that is *feasible*[1], then a queue-length based *maximum-weight* policy will serve data while keeping the backlog in the system finite. This work has led to many exciting results ever since. However, there is no simple polynomial time scheme that determines the feasibility of a potential link-rate allocation.

Unlike wireline networks, in wireless networks the link capacities change depending on the schedules. This means that it is important to develop algorithms that operate across layers: algorithms should perform the routing, scheduling and power control in a joint manner. Motivated by this, there has been a recent interest in developing cross-layer optimization algorithms (see, for example [5], [13], [14], [19], [21]) for wireless networks. On the other hand, the recent success of congestion control in wireline networks has led researchers to incorporate congestion control into cross-layer optimization. [3], [11], [14], [17], [20], [22].

Most of the above joint cross-layer optimization problems have been shown to exhibit a nice decoupling property (see, for example [11], [21]). More precisely, a cross-layer optimization problem can be decomposed into multiple sub-problems, where each sub-problem corresponds to a single layer. The layers couple with each other by means of appropriate *dual variables*, which can be thought of as the congestion prices of queue-lengths at the links in the network.

The crux of the above work is the following: the optimal cross-layer algorithm corresponds to solving a global optimization problem of the following form: given network G on n nodes V and edge-set (links) E,

$$\text{maximize} \sum_{e \in E} p_e \lambda_e$$
$$\text{subject to } \lambda = (\lambda_e) \in \Lambda,$$

where $\mathbf{p} = (p_e)$ are coupling parameters and Λ is the set of all feasible link-rate allocations.

Clearly, the above optimization problem requires an algorithm to check the feasibility of a given link-rate allocation, i.e. verifying whether given $\lambda = (\lambda_e) \in \Lambda$ or $\lambda \notin \Lambda$.

B. Our contributions

The only existing result so far on the rate feasibility problem, to the best of our knowledge, is a negative result

[1]That is, there exists a scheme that can serve the data while keeping the backlogs in the system finite under appropriate conditions on arrival processes.

of NP hardness. In this paper, we show that even though this problem is NP-hard, there could be relevant subclasses of the problem that arise in practice which can be tackled efficiently. Specifically, first we give an approximation solution to the problem of checking rate feasibility when the node density is bounded (defined precisely in later sections). This approximation algorithm is novel, and could be of independent interest for membership testing on other convex sets. We also show that the rate feasibility problem can be efficiently solved when the nodes are restricted to a fixed width slab. For instance, Intelligent Vehicle Highway Systems (IVHS) applications require communication between cars restricted to roads - something that is topologically similar to the slab abstraction where the width is fixed but the length can be unbounded. Similarly, commercial air-routes are essentially "roads in the sky" too. We also make certain practical assumptions on the interference models which will be described in detail in the following sections.

We organize the paper as follows: Section II describes the problem setting, models and the restrictive assumptions. Sections III and IV give details of the first and second results respectively.

C. Related work

The problem of feasible rate allocation in the context of node-exclusive interference model (matching model) has been studied by Hajek and Sasaki [8]. This problem has a polynomial time approximation scheme. The problem of feasible rate allocation for the 2-matching, i.e. with additional "secondary interference constraints", was shown to be NP-hard by Arikan [1]. To the best of our knowledge, this problem has not been studied in other contexts.

There are, however, a large body of works on scheduling in wireless network with interference constraints. In [2]–[4], [8], [11], [14], [19] authors consider the matching based scheduling problem. In [6], authors consider a closely related problem of determining capacity in ad hoc wireless network under secondary interference model (2-matching model) by means of finding maximum size 2-matching in a Geometric graph. Similar to the ideas of [6], authors in [16] consider the problem of finding maximum weighted K-matching in a geometric graph. While these approaches provide an *almost stable* schedule for wireless packet scheduling, they do not readily answer the question of feasibility of rate allocation to links in wireless network. We note that the approaches of [6] and [16] have been derived from that of [9]. In spirit, our methods are similar. But we note that neither of the above papers address question of rate feasibility, or equivalently membership in a convex set. We also note a similarity of an algorithm we propose here to the recent work [10] for finding stable scheduling algorithms with independent set constraints.

II. Model and results

A. Problem statement

A wireless network carrying a collection of single-hop, constant-bit-rate (CBR) flows, can be represented as an undirected graph $G = (V, E)$, where V represents the set of *nodes*, and $E \subseteq V \times V$ is a symmetric[2] relationship that represents the set of bi-directional *links*. We assume all links of the network have the same capacity, which is normalized to unity.

Interference among concurrent origin-to-destination communications would place constraints on the set of links that can be simultaneously active at any instant in time. Define *primary (node exclusive) interference constraints* as the interference caused by two links sharing a common node. The *secondary interference constraints* impose additional restrictions – two links (i, j) and (i', j') of E can not be active simultaneously if $(i, j') \in E$.

A collection of links can be represented using its appropriate indicator vector, whose dimension is equal to the total number of links in the network. The set of all possible indicator vectors corresponding to collections of non-conflicting links in the network can be assimilated to form a matrix $\mathbf{T}$. The number of columns of $\mathbf{T}$ will be exponential, in general.

A transmission *schedule* decides the set of non-conflicting simultaneous transmissions permitted at each given time instant. A single-hop rate-vector $\mathbf{r}$ is said to be *feasible* if there is a transmission schedule that achieves the prescribed rates for each link. The set of all feasible rate-vectors form the *single-hop capacity region* of a wireless network. We concern ourselves with the problem of deciding membership of an arbitrary rate-vector in the single-hop capacity region of a wireless network.

Let us suppose the MAC-layer of the wireless network uses a variable slot length TDMA scheme. Following [1], a rate vector λ belongs to the single-hop capacity region of a wireless network if and only if there exists $z \leq 1$, where

$$\begin{aligned} z = min\ \mathbf{1}^T\mathbf{x} \\ \mathbf{T}\mathbf{x} = \lambda, \end{aligned} \tag{1}$$

and $\mathbf{T}$ is a matrix whose columns represent collections of non-conflicting links. Even though the membership of a rate vector is expressible as an instance of a Linear Program (LP), since the number of columns of $\mathbf{T}$ can be exponential in terms of the links, it not guaranteed to have a tractable solution. The results in [1], [15] show that this membership problem is NP-hard.

Remark 1: Let Λ denote the collection of all feasible rate vectors, i.e. λ for which the equation (1) has a solution $z \leq 1$. We note the following *radial* property of the rate-region Λ: The set Λ is convex. Especially, $\mathbf{0} \in \Lambda$ and if $\lambda \in \Lambda$, then for any $\mu \in \mathbb{R}_+^n$ with $\mu \leq \lambda$ (component-wise), we have $\mu \in \Lambda$.

B. Summary of the restrictive models

We now clearly describe the network models for which we attain our results on the rate feasibility problem.

1) Conditions for the first result: Consider a wireless network of m nodes represented by $V = \{1, \ldots, m\}$ placed in a 2-dimensional geographic region in an *arbitrary* manner (not necessarily random) inside a $\sqrt{m} \times \sqrt{m}$ square of area

[2]That is, $(i, j) \in E \Leftrightarrow (j, i) \in E$.

m.[3] Let $E = \{(i, j) : i \text{ can transmit to } j\}$ be set of directed links between nodes indicating which nodes can communicate. Let $G = (V, E)$ be the directed graph capturing the network. We assume that the wireless network satisfies the following simple assumptions.

Assumption 1 (Bounded Transmission Radius): We assume that there is an $R > 0$ such that no two nodes that are at distance larger than R can establish a communication link with each other[4].

Assumption 2 (Bounded Density): Given node $v \in V$, let

$$B(v, R) = |\{u \in V : \ u \text{ is at distance at most } R \text{ from } v\}|.$$

We say that graph G has bounded density $D > 0$, if for all $v \in V$

$$\frac{B(v, R)}{R^2} \leq D, \text{ and } R^2 D = O(\log m).$$

We note that the above assumptions are fairly *mild*, in the sense that it is highly likely for any practically designed wireless network to satisfy these assumptions.

Example 1: Consider a geometric random graph of n nodes obtained by placing n nodes in the $\sqrt{n} \times \sqrt{n}$ square uniformly at random and connecting any two nodes that are within distance $r = \Theta(\sqrt{\log n})$ of each other. We denote this graph by $G(n, r)$. It is well known that $G(n, r)$ is connected with high probability for such an $r \geq r_c = \Theta(\sqrt{\log n})$. The proof of the below lemma is not given here due to space constraints.

Lemma 2: The $G(n, r)$ satisfies Assumptions 1-2 with high probability.

2) Conditions for the second result: The model of communication for our second result further distinguishes a *communication radius* r_c, and an *interference radius* r_i, with $r_i > r_c$.

Definition 1: Communication radius: Two nodes can establish a communication link if and only if they are separated by a distance less than the communication radius, r_c.

Definition 2: Interference radius: Two nodes have an interfering link if and only if they are separated by a distance less than the interference radius, r_i.

The assumption that $r_i > r_c$ is a very justifiable assumption and is routinely used in wireless literature. (e.g. [7])

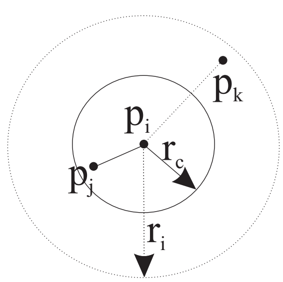

Fig. 1. An illustration of the *communication radius* and the *interference radius*. Nodes p_i and p_j (p_k) can communicate (interfere) with each other.

[3]Our algorithm actually works for arbitrarily placed nodes in the plane, but we consider the square just for presentation.

[4]this does not imply that nodes within distance R *must* communicate.

In figure 1, the broken line indicates an interference link, while the solid line denotes a communication link. Given an arrangement of points, this implicitly specifies the sets of all non-conflicting links according to the interference constraints. In other words, the secondary interference constraint with this distinction is that communication links (i, j) and (i', j') interfere a) if i, j, i', j' are not all distinct, or b) if i, j, i', j' distinct but (i, i') is a potential interference link, i.e. $d(i, i') < r_i$.

There are no assumptions of density for the second result. However, we place the nodes in $\mathcal{R}^2$ restricted to an area whose width is fixed (does not grow with the number of nodes).

III. First result

For ease of exposition, we will transform the *link based* rate feasibility problem with a total of n links (on m nodes) into a naturally suggested equivalent *node based* rate feasibility problem on n nodes where independent sets form the set of all the feasible simultaneous transmissions. The assumptions stated will transform equivalently. A precise justification for this has been relegated to section III-C as lemma 5 to prevent sidetracking of the discussion.

We will now proceed to presenting our main algorithm for the above node based rate feasibility setting in Section III-A in detail.

A. Algorithm: Node based rate feasibility

In this section, we will present a polynomial in n time approximation algorithm for checking feasibility of a node-based rate vector (of length n). Given $\varepsilon > 0$ and node-based rate vector λ,

1. If $\lambda \in (1 - \varepsilon)\Lambda$, the algorithm outputs YES and a time division such that
$$\lambda \leq \sum_{k=1}^{K(n)} \alpha_k I_k,$$
with $\alpha_k \geq 0, \sum_k \alpha_k \leq 1$ and $K(n) = \mathsf{poly}(n)$.
2. If $\lambda \notin (1 - \varepsilon)^{-1}\Lambda$, the algorithm outputs NO.
3. Otherwise (i.e. iff λ falls in the "ε-boundary" that was not covered in 1. or 2.), the answer can be ANYTHING.

1) An intuitive description of the Algorithm: Figure 2 shows nodes placed in a square in arbitrary manner. Suppose $R > 0$ is an upper bound on the distance between two nodes that can establish a communication link as in Assumption 1, and that the node placement satisfies the Assumption 2 with density bound being $O(\log n)$. We now present an intuitive description of the algorithm through the illustration shown in figure 3.

The algorithm involves multiple *iterations* of the following procedure. In each iteration, the plane is partitioned into a union of disjoint rectangles and a uniform-randomly positioned rigid grid-like *boundary region* whose thickness is R. This implies that nodes across distinct rectangles will always remain disconnected and hence union of independent sets from each rectangle will remain an independent set. The rate feasibility problem is then independently solved for each rectangular area using an exhaustive search. Because of the density assumptions and the nature of the partition, the size of each of the subproblems will be $O(\log n)$ making the exhaustive search for each subproblem polynomial in n. Failure of rate feasibility in a subgraph clearly implies infeasibility as our answer. If not, we get feasible schedules from each rectangle which can then be *merged* in a natural way to get a global schedule. This global schedule in each iteration will satisfy the rate feasibility on all nodes except for those that fall in the *boundary region*. However, if we ensure that each node has a small probability of falling in the boundary region at any given iteration, in a sufficiently large number of independent iterations each node will fall into the boundary for a small fraction of times with high probability. Hence an *averaged* schedule over all these iterations will be a close estimate to λ, the query rate vector.

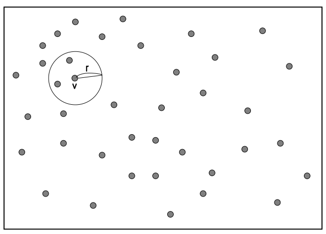

Fig. 2. We consider wireless communication in a Euclidean plane. Nodes are placed in an arbitrary manner such that at each point of the plane, node density is bounded by $O(\log n)$. And there is a constant radius r, so that two nodes can communicate and interfere each other only if the distance between them is less than r.

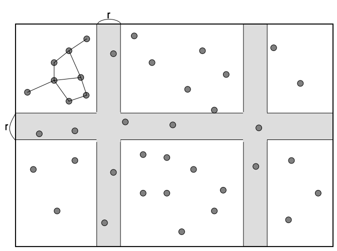

Fig. 3. Our algorithm for checking feasibility randomly defines the cross-road shaped regions indicated above in grey. In each of the components of the non-boundary (white) regions, the rate feasibility is checked using a standard linear programming formulation. This can be accomplished in polynomial time as each component has O(log n) nodes. The results from the various components are merged to obtain a time-division scheme for the entire graph. The confidence of the procedure can be improved by appropriate repetition of the above procedure.

2) Algorithm: We now present the precise details and the pseudo-code for the algorithm. In the pseudo-code below, LP is the subroutine used for computing an exhaustive solution to the rate feasibility problem on the rectangles created by the random partition. MERGE is the subroutine that takes schedules over distinct rectangles and merges them to create our global schedule Algorithm FC (Feasibility Check) which seeks to determine the feasibility of the node-based rate vector λ.

Procedure FC (λ)

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(1) Let $K(\varepsilon) = \frac{4eR}{\varepsilon}$ and $L(\varepsilon) = \frac{3\log n}{\varepsilon}$.

(2) Algorithm runs for $L(\varepsilon)$ iterations. Begin with iteration $\ell = 0$.

(3) For $\ell \leq L(\varepsilon)$, do the following:

  (a) Choose two numbers a and b uniformly at random from $[0, K(\varepsilon)]$.

  (b) Obtain boundaries:

    (i) Given a and b, a point (x, y) in the square is said to be a boundary point if $x \in [a + jK(\varepsilon), a + R + jK(\varepsilon)]$ or $y \in [b + jK(\varepsilon), b + R + jK(\varepsilon)]$ for some $j \in \mathbb{Z}$.

    (ii) Let B be the set of all such boundary points.

  (c) Obtain partition:

    (i) Obtain sub-graph G' from G by removing all the edges that have an end vertex in B.

    (ii) This creates a partition of graph G' into disconnected components $S_1, \ldots, S_m$ for some m. The components are such that no two nodes in the same component are at distance more than $\sqrt{2}K$. Under assumptions 1-2, each component has $O(\log n)$ nodes.

  (d) Obtain an estimate:

    (i) For each component S_i, $1 \leq i \leq m$, execute LP to check the feasibility of λ restricted to the nodes of S_i.

    (ii) If on any component, algorithm LP returns not feasible, then declare NOT FEASIBLE and stop.

    (iii) Else, obtain the decomposition of λ restricted to each component S_i. Say this is given by the vector $((\alpha_k^i, I_k^i)_k)$ of polynomial in n length, where α_k^i's are real numbers and I_k^i's are independent sets of S_i. That is, let λ^i be restriction of λ to nodes of S_i. Then
$$\lambda^i \leq \sum_k \alpha_k^i I_k^i, \quad \text{such that} \sum_k \alpha_k^i \leq 1.$$

    (iv) Merge the $((\alpha_k^i, I_k^i))_{1\leq i\leq m}$ according to a procedure **MERGE** to obtain $(\beta_k, I_k)_k$ such that I_k are independent sets of G and $\beta_k \geq 0$, $\sum_k \beta_k \leq 1$. Denote $T(\ell)$ as this decomposition of ℓ^{th} iteration $(\beta_k, I_k)_k$.

  (e) Set $\ell = \ell + 1$ and repeat.

(4) Declare the final estimate of decomposition of λ as $\hat{T}$ which is obtained by the following longer normalized vector
$$\left(\frac{1}{L(\varepsilon)} T(\ell)\right)_{1\leq \ell \leq L(\varepsilon)}.$$

Note that in each iteration of FC , any two nodes of G that are in different components of G' do not share an edge of G. So the subset of nodes of G obtained by merging independent sets for each component of G' is an independent set of G.

3) Subroutine MERGE: The following is a pseudo-code for the sub-routine **MERGE** that is used in FC to obtain convex combination of independent sets for the whole graph G from those of each component of G'.

MERGE

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(1) Given $((\alpha_k^i, I_k^i))_{1\le i\le m, 1\le k\le k_i}$, for each $1 \le i \le m$ and $0 \le j \le k_i$, let $\gamma_j^i = \sum_{k=1}^{j} \alpha_k^i$ (let $\gamma_0^i = 0$). Let $U_i = \{\gamma_0^i, \gamma_1^i, \gamma_2^i \ldots, \gamma_{k_i}^i\}$, and $U = \bigcup_i U_i$. Let $0 = \beta_0 < \beta_1 < \beta_2 < \ldots < \beta_\ell$ be the elements of U.

(2) For each i and j such that $1 \le i \le m$ and $1 \le j \le \ell$, let $j(i)$ be the (unique) index such that $\gamma_{j(i)-1}^i \le \beta_{j-1} < \beta_j \le \gamma_{j(i)}^i$, and let $I_j = \bigcup_i I_{j(i)}^i$.

(3) Output $T = (\beta_1, I_1, \beta_2 - \beta_1, I_2, \beta_3 - \beta_2, I_3, \ldots, \beta_\ell - \beta_{\ell-1}, I_\ell)$.

Note that ℓ, the number of independent sets in the output of **MERGE** satisfies $\ell \le \sum_{i=1}^m k_i$. Since $m \le n$, if each k_i is polynomial in n, then so is ℓ.

4) Subroutine LP : In Algorithm FC , we used the following sub-routine LP to check the feasibility of λ restricted to each component in the graph partition. Algorithm LP runs on a given graph G_0 with m nodes and LP takes λ_0, a rate vector on G_0 as its input.

LP

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(1) Obtain all the possible independent sets of G_0. Let $I_1, \ldots, I_Q$ be those independent sets. Note that the number of independent sets $Q \le 2^m$, where m is number of nodes of G_0.

(2) Solve the following Linear Program :

$$\sum_{i=1}^{Q} x_i = 1, \text{ sub. to. } \lambda_0 \le \sum_i x_i I_i.$$

(3) If the above Linear Program has a solution, return $(I_1, x_1, I_2, x_2, \ldots, I_Q, x_Q) = (x_i, I_i)_{1\le i\le Q}$.

(4) Else, return NOT FEASIBLE.

B. Analysis

1) Correctness: We state the following theorem about the correctness of FC .

Theorem 3: Given a graph G that satisfies Assumptions 1-2 and an arbitrary $\varepsilon > 0$, the algorithm FC has the following properties:

(1) If $\lambda \in \Lambda$, then w.h.p. FC outputs a feasible time division scheme $\hat{T} = (\alpha_k, I_k)_{k\le M}$ with $M = \mathsf{poly}(n)$,

$$(1-\varepsilon)\lambda \le \sum_k \alpha_k I_k,$$

such that $\sum_k \alpha_k \le 1, \alpha_k \ge 0$.

(2) If $\lambda \notin (1-\varepsilon)^{-1}\Lambda$, then with high probability, FC outputs NOT FEASIBLE.

Proof:

We begin by proving property (1). Fix a node v of graph G. Recall that $L(\varepsilon) = \frac{3\log n}{\varepsilon}$ is the number of iterations of FC . Let X_i be the indicator random variable for the event that v is in boundary in the ith iteration of FC . Let $X = \sum_i X_i$. Then

$$E[X_i] \le \frac{2R}{K(\varepsilon)} = \frac{\varepsilon}{2e}$$

and hence

$$E[X] \le \frac{\varepsilon L(\varepsilon)}{2e}.$$

Note that X is Binomial random variable since X_i's are independent and identically distributed. Hence by standard application of Chernoff bound, we obtain

$$\mathbb{P}[X > \varepsilon L(\varepsilon)] < 2^{-\varepsilon L(\varepsilon)} = \frac{1}{n^3}.$$

Now using the union bound over all the nodes, the probability that all nodes of G will be in the boundary at most $\varepsilon L(\varepsilon)$ times during the $L(\varepsilon)$ iterations is at least $1 - \frac{1}{n^2}$. Since $\lambda \in \Lambda(G)$, for each connected component of G', there is at least one solution to the linear programming in LP . So the algorithm FC will output an estimate $\hat{T}$ and never output NOT FEASIBLE. Since the output $\hat{T}$ of FC is the average of the decompositions of $L(\varepsilon)$ many iterations and each node is not a part of the boundary for at least $1-\varepsilon$ fraction of iterations, we have $(1-\varepsilon)\lambda \le \lambda(T)$. Now, to see that $M = \mathsf{poly}(n)$, note that due to the fact that M is upper bounded by the summation of the number of independent sets of the output of LP for each connected component, we only need to show that the number of independent sets of the output of LP for each component is polynomial over n. Under the Assumptions 1-2, for any $\varepsilon > 0$, each component has $O(K^2(\varepsilon)D) = O(\frac{R^2 D}{\varepsilon^2}) = O(\log n)$ nodes. Hence, the number of independent sets in each component is polynomial over n, and which shows that $M = \mathsf{poly}(n)$.

Now, we present proof of part (2) as follows: Suppose that $(1-\varepsilon)\lambda \notin \Lambda$. As established above, the probability that all of the nodes of G will be in the boundary at most $\varepsilon L(\varepsilon)$ times during the $L(\varepsilon)$ iterations of FC with probability at least $1 - \frac{1}{n^2}$.

Now suppose property (2) does not hold. That is, for $\lambda \notin (1-\varepsilon)^{-1}\Lambda$, algorithm FC does not output NOT FEASIBLE. That is, algorithm FC returns decomposition $\hat{T}$ of FC . But as we argued above, since each node is not part of the boundary at least $(1-\varepsilon)$ fraction of the time, it must be the case that $(1-\varepsilon)\lambda \le \lambda(T)$. Since $\lambda(T) \in \Lambda$, we have by radial property of Λ as established in remark 1, $(1-\varepsilon)\lambda \in \Lambda$. This leads to a contradiction of our assumption. That is algorithm FC must generate NOT FEASIBLE for $\lambda \notin (1-\varepsilon)^{-1}\Lambda$ with probability at least $1 - \frac{1}{n^2}$. This completes the proof of Property (2) and that of Theorem 3. ■

2) Time complexity: In this section, we establish that the running time of algorithm FC is $\mathsf{poly}(n)$.

Theorem 4: Given a wireless graph G satisfying Assumptions 1-2 with transmission radius R and density bound D, the running time of FC is

$$O\left(\frac{n\log n 2^{O\left(\frac{R^2D}{\varepsilon^2}\right)}}{\varepsilon}\right).$$

Under Assumption 2, we have $R^2D = O(\log n)$. In that case, the running time of FC is $\mathsf{poly}(n)$. When $R^2D = O(1)$ the running time of FC is $O(n \log n)$.

Proof: The algorithm FC partitions the graph into disjoint components, then the LP is executed on each component followed by procedure MERGE on the schedules computed in each iteration. We compute the complexity of each of these three different sub-routines.

The partition requires a random selection of a and b. Then, each node has to check whether to retain an edge or not. This requires each node to perform a number of operations proportional to its degree which is $O(R^2D)$. Since there are n nodes, the total number of operations is $O(nR^2D)$.

Each component has $O\left(K^2(\varepsilon)D\right) = O\left(\frac{R^2D}{\varepsilon^2}\right)$ nodes. Hence, the number of independent sets in each component can be at most $2^{O(\frac{R^2D}{\varepsilon^2})}$. Hence, the LP for each component takes $2^{O(\frac{R^2D}{\varepsilon^2})}$ time by standard Ellipsoid algorithm for linear programming, and the total running time of the LP in one iteration of FC is $O\left(n2^{O(\frac{R^2D}{\varepsilon^2})}\right)$. It can be easily verified that the MERGE procedure takes $O\left(n2^{O(\frac{R^2D}{\varepsilon^2})}\right)$ time. Note that the size of merged output of FC is also $O\left(n2^{O(\frac{R^2D}{\varepsilon^2})}\right)$.

Thus, over a total of $O(\log n/\varepsilon)$ iterations, we find that the running time of FC is $O\left(\frac{n\log n 2^{O(\frac{R^2D}{\varepsilon^2})}}{\varepsilon}\right)$. ■

C. Algorithm: K-Matching interference Model

In this section, we justify the application of our algorithm to graphs satisfying assumptions 1-2 under K-matching interference constraints for a general K. For this, we show that on a graph satisfying assumptions 1-2, the problem of checking link rate feasibility under the 2-matching model is equivalent (by means of a polynomial time reduction) to checking node-based rate feasibility in a graph satisfying Assumptions 1-2 with independent set scheduling. The same reduction can then be extended for any K-matching problem for $K > 2$.

To this end, let nodes be placed in the square as before with graph $G = (V, E)$ satisfying assumptions 1-2. We construct a new graph, $\hat{G}$ called the *adjoint graph* as follows: $\hat{G}$ has a vertex corresponding to each edge in E. Let $\hat{V}$ denote this set. For edge $(i, j) \in E$, imagine that the vertex in $\hat{V}$ is placed at the position of i in the square (more than one vertex is allowed to be placed at the same position). Place an edge between two vertices of $\hat{V}$ if the corresponding edges had a conflict of simultaneous transmissions under the 2-matching model. $\hat{G}$ is undirected since conflict relations between the links are symmetric by definition. Note that by definition of $\hat{G}$, the interference constraints in G are now independent set constraints in $\hat{G}$ giving us a node based rate feasibility problem on $\hat{G}$. Now, we can use the algorithm FC on this interference graph. The above construction of interference graph takes at most $O(nR^2D) = O(n \log n)$ operations.

Lemma 5: Under assumptions 1 and 2 on G, the adjoint graph $\hat{G}$ under 2-matching model also satisfies the Assumptions 1 and 2 with a different $\hat{R} = O(R)$.

Proof: (outline) First, we note that Assumption 1 is satisfied on $\hat{G}$ corresponding to an $\hat{R} = 4R$ when Assumption 1 holds for G.

As for Assumption 2, first note that the nodes of $\hat{V}$ that are separated by more than $4R$ are disconnected. Since these nodes correspond to end points of edges, by using the fact that any area of $O(R^2)$ can be covered by $O(1)$ discs of radius R, and the Assumption 1 for original graph G implies that the desired property holds. ■

Similar construction can be performed for any K-matching problem for finite $K > 2$.

IV. Second result

The second result critically uses the results of Matsui [12] on fractional coloring of unit disk graphs in a fixed width slab. However, this also involves, what could be of independent interest, a necessary non trivial generalization of the applicability of the algorithms of [12] to a more general class of graphs that we will define. This makes it possible to apply this generalized form of Matsui's results to wireless networks. The conditions and the model on the wireless network for the second result have already been precisely specified in II-B2. We maintain the notation used in [12] here. Next, we start off giving a brief overview.

A. Intuitive summary

First, we note that the fractional coloring problem on a graph is analogous to the *node based* rate feasibility problem that corresponds to asking for an *equal* rate at each node. Matsui's algorithms solve the fractional coloring problem on *unit disk* graphs for nodes in a fixed width slab. Our first observation is to note that this procedure can be generalized for any given asking rate - which is precisely the *node-based* rate feasibility problem.

Now, although our *link-based* rate feasibility problem can be naturally transformed to a node-based rate feasibility problem on its *adjoint graph*, this adjoint graph clearly needs to be a unit disk graph to apply Matsui's algorithm. For this, the adjoint needs to be produced as a graph with locations specified for its nodes (the original links) in the plane. If this is done, for instance, by assigning each link its midpoint, no assumptions on the original wireless network graph might actually produce an adjoint that can be shown to be isomorphic to a unit disk graph. The reason for this is that we can easily have a situation for the secondary interference model where two links conflict while some other pair of links that are actually "closer" do not conflict.

This is where the second observation on the results of Matsui becomes crucial. Rather than trying to find an adjoint that fits into the unit disk model, we note that the results for unit disk graphs in fact can be generalized to a class of graphs that we call the (d_{min}, d_{max}) graphs. These are graphs that satisfy the property for two appropriate constants d_{min} and d_{max} that: (1) two nodes must be connected if separated by less than d_{min} and (2) two nodes must be disconnected if separated by more than d_{max}. In other words, the observation amounts to the fact that all we need for Matsui's algorithm to work is that nodes that are too close must have an edge while those that are too far can not have an edge (while not caring about what happens between d_{min} and d_{max}). Thus, we do not necessarily need the unit disk graph structure which is much more stringent, and in fact, fails for the adjoints we can define later on.

Once this generalization is shown, we define an adjoint easily and show that it is a (d_{min}, d_{max}) graph for d_{min} and d_{max} which depend on the interference and communication radii r_I and r_C. This implies that the generalized form of Matsui's algorithm on the (d_{min}, d_{max}) graphs can be used to solve the rate feasibility problem for a wireless network constrained to a fixed-width slab. We now begin with a brief review of fractional coloring on unit disk graphs [12].

B. Unit Disk Graphs on Fixed-Width Slabs

The *unit disk graph* defined by a point-set $P \subseteq \mathcal{R}^2$, is an undirected graph $G(P) = (P, E)$, where $E \subseteq P \times P$, and $(\mathbf{p}_1, \mathbf{p}_2) \in E \Leftrightarrow \|\mathbf{p}_1 - \mathbf{p}_2\|_2 \leq 1$, where $\| \bullet \|_2$ represents the 2-norm of the vector argument. The *unit sphere graph* induced by a point-set $P \subseteq \mathcal{R}^3$ in three dimensions can be defined similarly.

Using a lexicographic ordering of the vertex set V, each independent set of $G = (V, E)$ can be represented by a $|V|$-dimensional indicator vector, whose i-th entry is set to unity if and only if the vertex v_i belongs to the independent set. The indicator vectors of all the independent sets of a graph can be assimilated to form a matrix $\mathbf{M}$. Following [12], the *fractional coloring problem* is defined as

$$\begin{aligned} z = min\ \mathbf{1}^T\mathbf{x} \\ \mathbf{Mx} \geq \mathbf{1} \\ \mathbf{x} \geq \mathbf{0} \end{aligned} \tag{2}$$

Since there can be a large number of columns in $\mathbf{M}$, the above LP does not yield a tractable solution in the general case. [12] contains a polynomial time algorithm for the fractional coloring of unit disk graphs whose vertices lie in a fixed width slab $S_k = \{(x, y) \in \mathcal{R}^2 \mid 0 \leq y \leq k\}$. We now present the relevant details of this procedure.

Let $\widehat{P} \subseteq P$ be a subset of the point-set P that defines the vertices of a unit disk graph, we define

$$\min\{\widehat{P}\} := \min\{\lfloor x \rfloor \ \mid (x, y) \in \widehat{P}\}$$

and

$$\mathcal{B}(P) := \{\widehat{P} \subseteq P \mid \widehat{P} \text{ is an independent set of } G(P)$$
$$\text{and } \forall (x, y) \in \widehat{P}, \lfloor x \rfloor = \min\{\widehat{P}\}\}$$

Given a point-set $P \subseteq \mathcal{R}^2$, and its associated unit disk graph $G(P)$, its *auxiliary graph* $A(P)$ is a directed graph with $\{s, t\} \cup \mathcal{B}(P)$ as its vertex set, while the directed set of edges are given by

$$\begin{gathered} \{(s, \widehat{P}) \mid \forall \widehat{P} \in \mathcal{B}(P)\} \cup \{(\widehat{P}, t) \mid \forall \widehat{P} \in \mathcal{B}(P)\} \cup \\ \{(\widehat{P}, \widetilde{P}) \in \mathcal{B}(P) \times \mathcal{B}(P) \mid \min\{\widehat{P}\} < \min\{\widetilde{P}\}, \\ \text{and } \widehat{P} \cup \widetilde{P} \text{ is an independent set of } G(P).\} \end{gathered}$$

There is a one-to-one correspondence between the family of independent sets in $G(P)$ and the set of $(s-t)$-paths in $A(P)$. The maximum independent set corresponds to the longest $(s-t)$-path in $A(P)$. When the point-set $P \subseteq S_k$, where $S_k = \{(x, y) \in \mathcal{R}^2 \mid 0 \leq y \leq k\}$ represents a slab of width k, is is shown in reference [12] that $|\mathcal{B}(P)| = O(n^{2\lceil \frac{2k}{\sqrt{3}} \rceil})$ where $n = |P|$. The same reference also presents a procedure by which the longest $(s-t)$-path in $A(P)$ can be found in $O(n^{4\lceil \frac{2k}{\sqrt{3}} \rceil})$ time. The fractional coloring problem in equation 2 reduces to the following flow-problem on $A(P)$, where $\mathbf{x}$ is a vector of variables indexed by the set of arcs in $A(P)$, $\delta^+(v)$ ($\delta^-(v)$) is the set of arcs entering (leaving) the vertex v in $A(P)$,

$$\begin{gathered} min\ \textstyle\sum_{e \in \delta^+(s)} x_e \\ \text{subject to:} \\ \textstyle\sum_{e \in \delta^+(v)} x_e - \sum_{f \in \delta^-(v)} x_f = 0, \forall v \in \mathcal{B}(P) \\ \textstyle\sum_{v \in \{\widehat{P} \in \mathcal{B}(P) | p \in \widehat{P}\}} \sum_{e \in \delta^+(v)} x_e \geq 1, \forall p \in P \\ \mathbf{x} \geq \mathbf{0}. \end{gathered} \tag{3}$$

The above flow-problem can be solved in strongly polynomial time with respect to $|\mathcal{B}(P)|$ ([18]). The optimal flow can then be decomposed into non-negative combinations of $(s-t)$-paths in $A(P)$. Each $(s-t)$-path in $A(P)$, which corresponds to an independent set of $G(P)$, identifies a column in the $\mathbf{M}$ matrix in equation 2, and the solution to the fractional coloring problem for unit disk graphs whose point-set vertices lie in a slab of fixed width k can be identified in strong polynomial time.

C. Generalizing unit disk graph results to a larger class

We first show that the results on the fractional coloring problem in [12] can be extended to a more general class of graphs $G(P) = (P, E)$ induced by point sets $P \subseteq \{(x, y) \in \mathcal{R}^2 \mid 0 \leq y \leq k\}$.

Definition 3: Let us define a (d_{min}, d_{max}) graph as any graph that can be induced by a point set $P \subseteq \{(x, y) \in \mathcal{R}^2$ that satisfies the following two properties for strictly positive finite constants d_{min} and d_{max}:

1) *Property P1*:
$$\forall \mathbf{p}_1, \mathbf{p}_2 \in P, \|\mathbf{p}_1 - \mathbf{p}_2\| \geq d_{max} \Rightarrow (\mathbf{p}_1, \mathbf{p}_2) \notin E \tag{4}$$
2) *Property P2*:
$$\|\mathbf{p}_1 - \mathbf{p}_2\| < d_{min} \Rightarrow (\mathbf{p}_1, \mathbf{p}_2) \in E \tag{5}$$

That is, there is an (no) edge connecting two points $\mathbf{p}_1$ and $\mathbf{p}_2$ in $G(P)$ separated by a distance less than (larger than) or equal to d_{min} (d_{max}). There might, or, might not be an edge between two points whose distance of separation is between d_{min} and d_{max}, however. We note the class of unit disk graphs for which the results in [12] have been shown is equivalent to setting a $d_{max} = d_{min}$ in the above more general model. The intuitive idea is that the necessity for properties 1 and 2 occurs independently and hence we would still be able to solve the problem even for two different constants in the two properties.

Theorem 6: The results in [12] hold for the class of (d_{min}, d_{max}) graphs.

Proof: (outline) The extensions to the various arguments stated earlier on unit disk graphs require the following definitions. If $\widehat{P} \subseteq P$, then

$$\min\{\widehat{P}\} := \min\left\{\left\lfloor \frac{x}{d_{max}} \right\rfloor\right\}$$

and

$$B(P) := \{\widehat{P} \subseteq P \mid \widehat{P} \text{ is an independent set of } G(P), \\ \text{and } \forall (x,y) \in \widehat{P}, \left\lfloor \frac{x}{d_{max}} \right\rfloor = \min\{\widehat{P}\}\}$$

The definition of the auxiliary graph $A(P)$ is similar to the one in [12]. That is, it is a directed graph with a vertex set $\{s,t\} \cup \mathcal{B}(P)$, and the directed set of edges are given by

$$\{(s,\widehat{P}) \mid \forall \widehat{P} \in \mathcal{B}(P)\} \cup \{(\widehat{P},t) \mid \forall \widehat{P} \in \mathcal{B}(P)\} \cup \\ \{(\widehat{P},\widetilde{P}) \in \mathcal{B}(P) \times \mathcal{B}(P) \mid \min\{\widehat{P}\} < \min\{\widetilde{P}\}, \quad (6) \\ \text{and } \widehat{P} \cup \widetilde{P} \text{ is an independent set of } G(P).\} \quad (7)$$

First, we verify that the directed graph $A(P)$ is transitive. Transitivity is necessary to ensure a one to one correspondence between the independent sets in the original graph to the paths in the auxiliary graph. To see how transitivity continues to hold, note that if $(\widehat{P}_1, \widehat{P}_2)$ and $(\widehat{P}_2, \widehat{P}_3)$ $(\widehat{P}_i \subseteq P, i = 1,2,3)$ are directed edges in $A(P)$, then $\min\{\widehat{P}_1\} < \min\{\widehat{P}_3\}$. Since the distance between members of $\widehat{P}_1$ and $\widehat{P}_3$ is at least d_{max}, and $\widehat{P}_1$ and $\widehat{P}_3$ are independent sets of $G(P)$, it follows that $\widehat{P}_1 \cup \widehat{P}_3$ is an independent set of $G(P)$. The transitivity property involving vertices $\{s,t\}$ follows directly from the definition of the edge set of $A(P)$. That is, if $(s,\widehat{P}_1)$ ($(\widehat{P}_1,\widehat{P}_2)$, $(s,\widehat{P}_1)$, respectively) and $(\widehat{P}_1,\widehat{P}_3)$ ($(\widehat{P}_2,t)$, $(\widehat{P}_1,t)$, respectively) are directed edges of $A(P)$, then $(s,\widehat{P}_3)$ ($(\widehat{P}_1,t)$, (s,t), respectively) is also a directed edge of $A(P)$.

Thus far, we only know that the flow problem is solvable in time polynomial only with respect to $|\mathcal{B}(P)|$. To show that $|\mathcal{B}(P)|$ is polynomial in $|P|$, we can upper bound $|\mathcal{B}(P)|$ as $O(|P|^{\alpha})$ where α is any upper bound for the size of an 'independent block' (a set belonging to $\mathcal{B}(P)$). Paralleling lemma 2 [12] we note that the region $[0, d_{max}) \times [0,k)$ can be covered using $\lceil 2d_{max}/d_{min} \rceil \lceil 2k/\sqrt{3}d_{min} \rceil$copies of the rectangle, $\{(x,y) | 0 \leq y \leq \sqrt{3}d_{min}/2, 0 \leq x \leq d_{min}/2\}$. Since each rectangle has a diagonal of length d_{min}, an independent block will have at most one point from each such rectangle and hence we have a constant upper bound, $\lceil 2d_{max}/d_{min} \rceil \lceil 2k/\sqrt{3}d_{min} \rceil$ on α. ∎

D. Rate feasibility problem on constrained wireless networks

We now address the problem of deciding membership in the capacity region of a planar wireless network whose nodes lie on a slab of width k. Suppose the point-set $P \subset \{(x,y) \in \mathcal{R}^2 \mid 0 \leq y \leq k\}$ represents the location of $|P| = n$ nodes of a wireless network, with the secondary interference model corresponding to radii of communication and interference, r_c and r_i as described previously. The link between $\mathbf{p}_1$ and $\mathbf{p}_2$ can be assigned two orientations – $(\mathbf{p}_1, \mathbf{p}_2)$ $(\mathbf{p}_2, \mathbf{p}_1)$, representing the situation where $\mathbf{p}_1$ $(\mathbf{p}_2)$ is the origin node and $\mathbf{p}_2$ $(\mathbf{p}_1)$ is the destination node. With an intention of defining an adjoint graph whose nodes correspond to directed edges in $G(P)$, we locate two members of the adjoint point-set, denoted by $(\mathbf{p}_1, \mathbf{p}_2)$ and $(\mathbf{p}_2, \mathbf{p}_1)$, at the mid-point between $\mathbf{p}_1$ and $\mathbf{p}_2$. This process is repeated for all communicating pairs in P. There is an edge between two vertices $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$ in the adjoint graph if and only if the communication between source $\mathbf{p}_i$ and destination $\mathbf{p}_j$ interferes with that between source $\mathbf{p}_k$ and destination $\mathbf{p}_l$. Let $\widehat{G}(P) = (Q, \widehat{E})$ denote the adjoint graph. We now show that $\widehat{G}(P)$ fits into the general model corresponding to properties P1 (equation 4) and P2 (equation 5).

Observation 7: For the adjoint graph $\widehat{G}(P)$ described above, there cannot be an edge between vertices $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$ if $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 \geq r_c + r_i$

Proof: Consider the various interference possibilities according to the primary (secondary) interference model and the location of the corresponding vertices of the adjoint graph shown in figure 4 (5). The length of the solid lines (corresponding to communicating links) is at most r_c while the length of the broken lines (corresponding to the interfering links) is at most r_i. For the primary constraints, $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 < r_c$. For the secondary interference, we see that $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 < r_c + r_i$. Hence, in either case, $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 < r_c + r_i$ if there is an edge between $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$. ∎

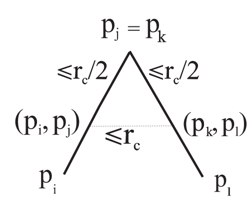

Fig. 4. Primary interference constraint and its effect on the separation distance of the mid-points of the communicating links.

Observation 8: For the adjoint graph $\widehat{G}(P)$ and the secondary interference model described above, there will be an edge between vertices $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$ if $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 < r_i - r_c$

Proof: Assume the modes $\mathbf{p}_i, \mathbf{p}_j, \mathbf{p}_j$ and $\mathbf{p}_l$, are all distinct and $\|(\mathbf{p}_i, \mathbf{p}_j) - (\mathbf{p}_k, \mathbf{p}_l)\|_2 < r_i - r_c$. Now, consider the secondary interference constraint as shown in the figure 6. According to the assumptions, $\|\mathbf{p}_i - \mathbf{p}_l\|_2 < r_c/2 + r_i - r_c + r_c/2 = r_i$ which means, the two links will interfere under

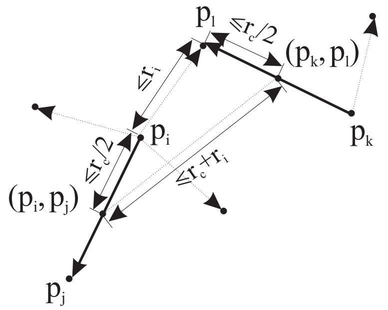

Fig. 5. Secondary interference constraint and its effect on the separation distance of the mid-points of the communicating links.

the interference constraints assumed and there will be an edge between $(\mathbf{p}_i, \mathbf{p}_j)$ and $(\mathbf{p}_k, \mathbf{p}_l)$ in the adjoint graph, $\widehat{G}(P)$. ■

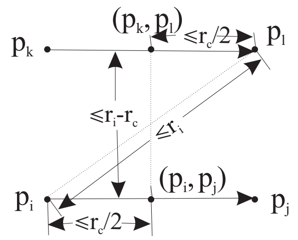

Fig. 6. An illustration in aid of the proof of Observation 8.

From observations 7 and 8 we see that the adjoint graph satisfies properties P1 (equation 4) and P2 (equation 5). The auxiliary graph $\widehat{A}(Q)$ of the adjoint graph $\widehat{G}(P) = (Q, \widehat{E})$ can be constructed using the vertex set $\{s, t\} \cup \mathcal{B}(Q)$, and edge set as described by equation 7. Let $\lambda_q (q \in Q)$ denote the component of an arbitrary rate-vector, whose entries are indexed by vertices of the adjoint graph $\widehat{G}(P) = (Q, \widehat{E})$. The following LP defined on $\widehat{A}(Q)$ determines the feasibility of λ:

$$\begin{array}{c} min \ \sum_{e \in \delta^+(s)} x_e \\ \text{subject to:} \\ \sum_{e \in \delta^+(v)} x_e - \sum_{f \in \delta^-(v)} x_f = 0, \forall v \in \mathcal{B}(Q) \\ \sum_{v \in \{\widehat{P} \in \mathcal{B}(P) | p \in \widehat{P}\}} \sum_{e \in \delta^+(v)} x_e \geq \lambda_q, \forall q \in Q \\ \mathbf{x} \geq \mathbf{0}. \end{array} \tag{8}$$

The flow-problem defined above can be solved in strong polynomial time. The following theorem is the decision procedure for the membership of λ in the single-hop capacity region of a wireless network.

Theorem 9: Let λ be an arbitrary rate vector whose entries are indexed by the vertices in the adjoint graph $\widehat{G}(P) = (Q, \widehat{E})$ and let $\mathbf{x}$ be the optimal flow identified by equation 8, λ is feasible $G(P)$ if and only if $\mathbf{1}^T \mathbf{x} \leq 1$.

From theorem 9 we have the fact that membership in the single-hop capacity region of a wireless network whose nodes are distributed in a plane of fixed width can be decided in strong polynomial time.

V. Conclusion

We have presented two restricted instances where the problem of rate feasibility in wireless networks has polynomial time algorithms, even though the most general problem version is NP-hard. Our results suggest that many important "practical" problem instances seem tractable for feasible rate allocation. Another potentially interesting aspect is that ideally, one would also like to have a distributed implementation for such algorithms. In that sense, the next task would be to design simple algorithms that can be incorporated seamlessly in cross-layer design. In summary, we believe that the results of this paper will have impact on the future design of algorithms for cross-layer design.

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